



# A New Key Rank Estimation Method to Investigate Dependent Key Lists of Side-Channel Attacks

Shuang Wang, Yang Li\*, Jian Wang

[li.yang@nuaa.edu.cn](mailto:li.yang@nuaa.edu.cn), [li.yangheu@gmail.com](mailto:li.yangheu@gmail.com)

Nanjing University of Aeronautics and Astronautics, Nanjing, China

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# Outline

- 1. Background
  - Side-channel attacks, divide and conquer
  - Key enumeration and key rank estimation
- 2. Motivation & Problems Setting
  - Key recovery vs key difference recovery
  - Dependent key list
- 3. Our Solution
  - Basic idea
  - Dependent-Key List Rank Estimation Algorithm (DK-REA)
- 4. Experimental Verification
  - Setup & Results & Analysis
- 5. Conclusions and Future Work

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# Side-channel attack and its evaluation

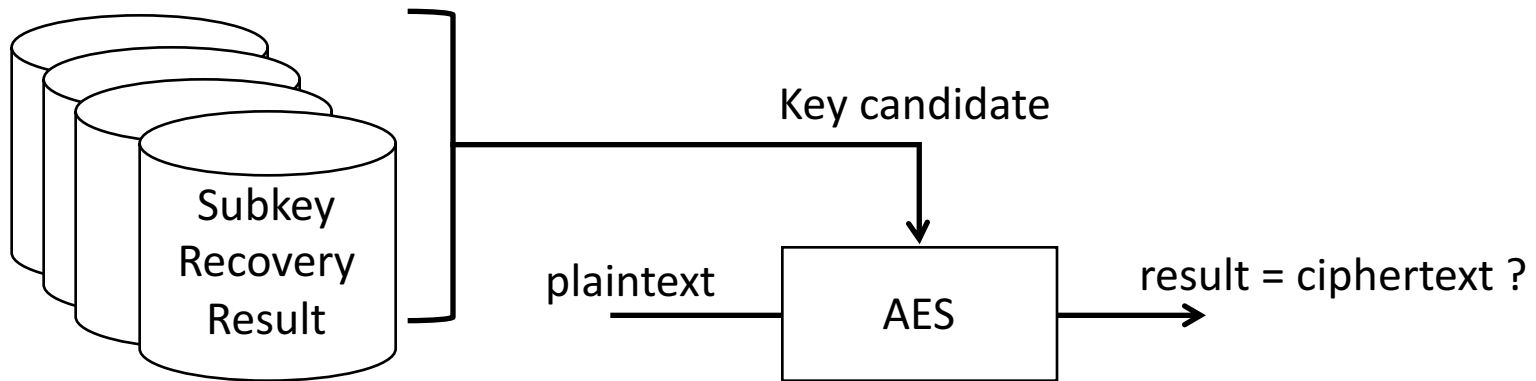
- Side-channel attack is powerful
  - Power analysis, time analysis, electromagnetic analysis etc.
  - Practical security threat to cryptographic devices
- SCA follows a divide-and-conquer approach
  - Key is divided into sub-keys to be recovered independently
  - When all sub-keys are successfully recovered, the whole key is recovered as well
  - Successful recovery means correct sub-key has “highest score” among all key candidates
- Security evaluation of cryptographic implementations
  - Data complexity (1k ~ 5M traces for key recovery)
  - Computational complexity (Usually not very high)

# How to define a successful attack

- **Full-key recovery**
  - Each sub-key has the highest score
  - Useful for practical attackers
  - Focus of this paper
- **Sub-key recovery**
  - Recover some sub-keys
  - Useful for security evaluation
    - Global Success Rate > 80% used in DPA contest v2
- **Existence of sensitive leakage**
  - Detection of leakage of sensitive information
    - T-test etc.
  - Useful for strict security evaluation, countermeasure proposal

# Trade-off between data and computations

- In full-key recovery, attackers can always use **plaintext-ciphertext test** to verify the correctness of a key

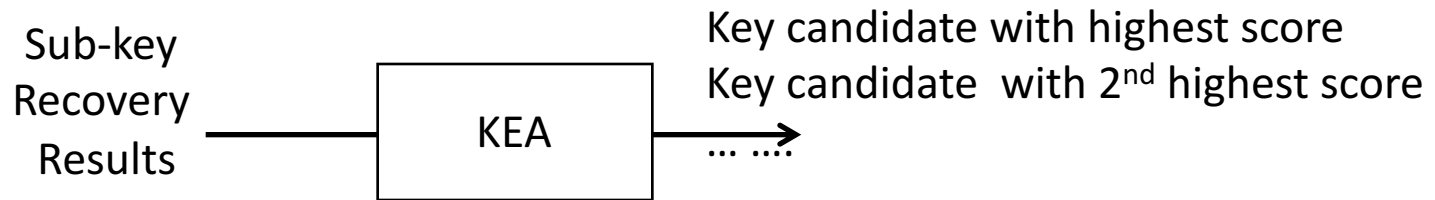


- E.g, verify  $2^{40}$  most likely key candidates to find the correct key
  - **More computation to trade for less data complexity!**
- How to enumerate the most likely keys from sub-key recovery results?
  - to minimize # of test trails.
- How to evaluate the rank of a certain key candidate?
  - to quickly understand required # of test trails.

# Key Enumeration Algorithm (KEA) and Key Rank Estimation Algorithm (REA)

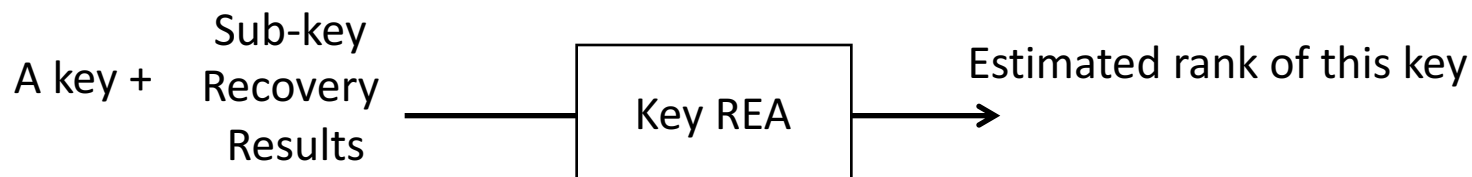
- Key Enumeration Algorithm

- Enumerate key candidates in decreasing order of likelihood
- Useful for practical attack



- Key Rank Estimation Algorithm

- Estimate rank of a given key
- Useful for known-key evaluation
  - Rank can be beyond the KEA's computational power



# Existing Key RE Algorithms

- First rank estimation algorithm proposed in Eurocrypt 2013
  - transfers search problem to a multidimensional problem
  - 10 bits of accuracy
- In 2015, Bernstein et al. introduced two better RE algorithms
  - First Alg. add post processing to increase accuracy from 10 bits to 5 bits
  - Second Alg. ranks the keys based on polynomial problem, flexible accuracy at the expense of run time.
- In 2015, Glowacz et al. proposed a rank estimation algorithm
  - Based on convolution of histograms.
  - Bounds with less than 1 bit of tightness within seconds of computation
  - Scales to larger key sizes
- In 2015, Martin et al. mapped the rank estimation problem
  - to a multi-dimensional knapsack problem
- All of them focused on merging independent key lists



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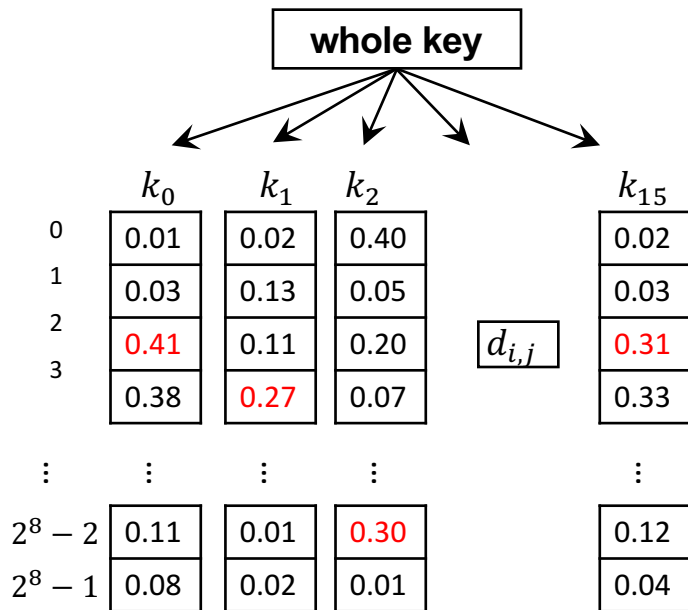
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# Motivation of this work

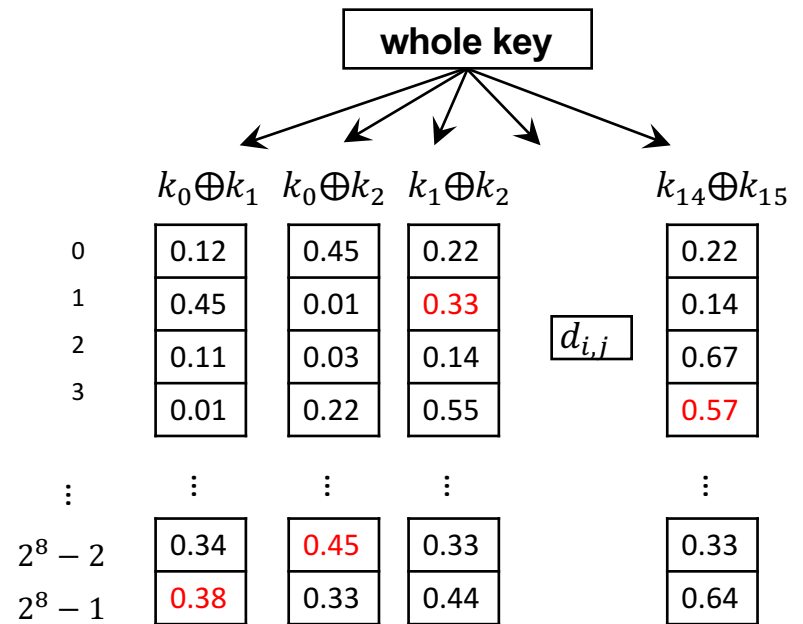
- Key is divided into sub-keys to be recovered independently
- But key can be divided in different formats, e.g. there are two well-known types of SCAs
- Key Recovery Attacks:
  - Target:  $k_0, k_1, \dots, k_{15}$
  - Based on leakage model from small component (e.g. S-box)
  - Differential Power Analysis, Correlation Power Analysis, Mutual Information Analysis, etc
- Key Difference Recovery Attack
  - Target :  $k_{0,1} = k_0 \oplus k_1, k_{0,2}, \dots, k_{0,15}, k_{1,2}, \dots, k_{1,15}, \dots, k_{14,15}$
  - Based on similarity of leakage from two small components
  - Collision attacks, Correlation-Enhanced Collision Attacks

# Can we combine these attack results?

## Key-recovery attack



## Key-difference recovery attack



Combination of  
dependent key lists

# Motivation

- Why to do so?
  - Intuitively, more score lists should lead to better attack result!
  - Different attacks use different information to recover key
    - Leakage model
    - Collision model
- We want to
  - Propose a key RE algorithm for dependent key lists (DK-REA),
  - Verify whether using more key lists is helpful

$k_0$	$k_1$	$k_2$	.....	$k_{15}$
0.13	0.14	0.14		0.29
0.34	0.22	0.22		0.31
...	...	...		...
0.24	0.45	0.45		0.68
	$k_{0,1}$	$k_{0,2}$	.....	$k_{0,15}$
	0.34	0.62		0.21
	0.27	0.43		0.35
	...	...		...
	0.15	0.71		0.77
		$k_{1,2}$	.....	$k_{1,15}$
		0.32		0.33
		0.77		0.15
		...		...
		0.15		0.36

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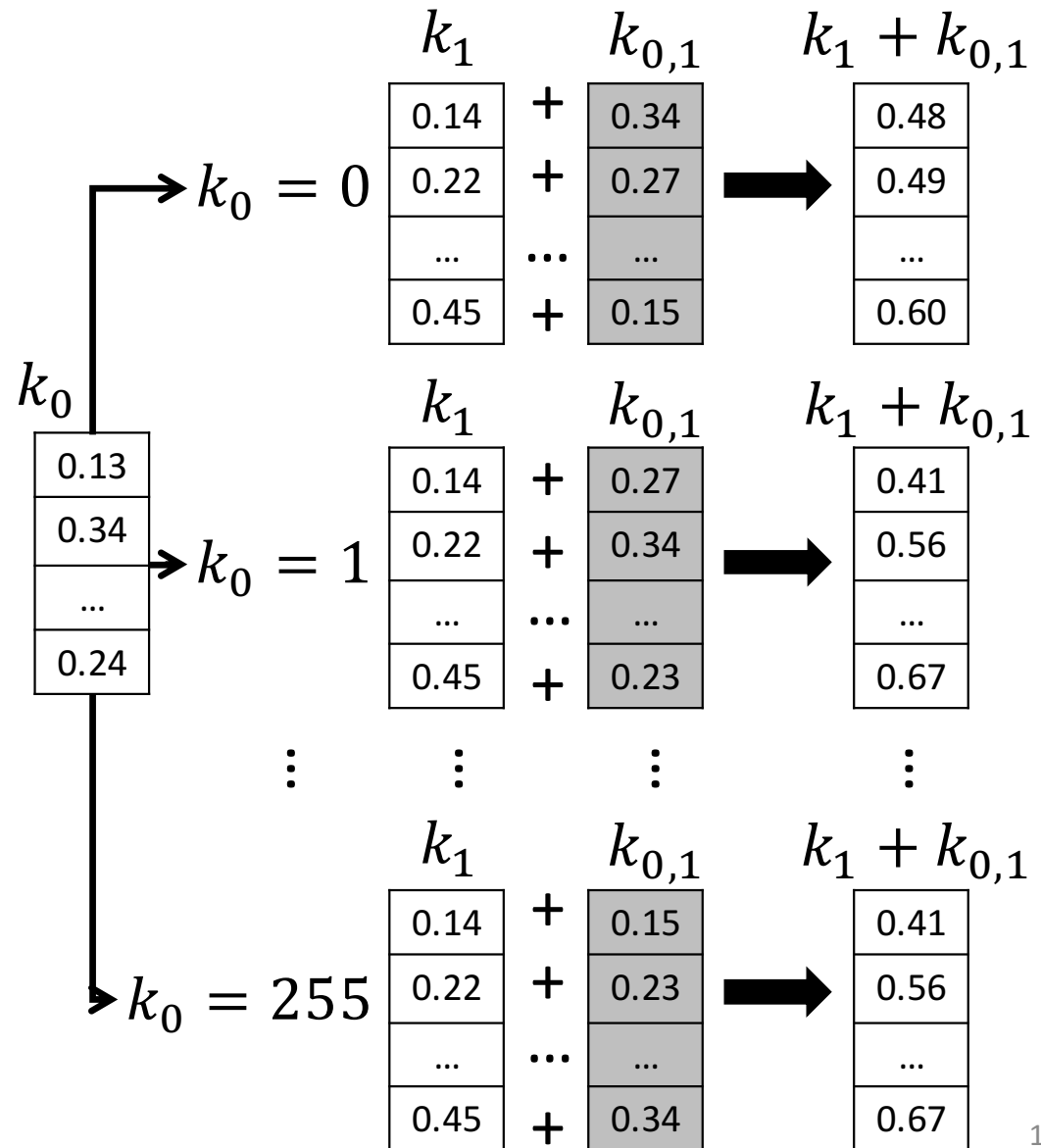
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# Straightforward solution cannot work

- DK-KRA is not trivial due to dependency
- Many meaningless combinations don't satisfy dependency
  - E.g.  $k_0 = 0, k_1 = 1, k_{0,1} = 2$  is invalid since  $k_0 \oplus k_1 \neq k_{0,1}$
  - Actually, most of combinations are invalid consider 16 key bytes and 15 key byte differences
  - Straightforward combination is impracticable
- A new method to combine dependent key lists
  - Deal with dependency before combination

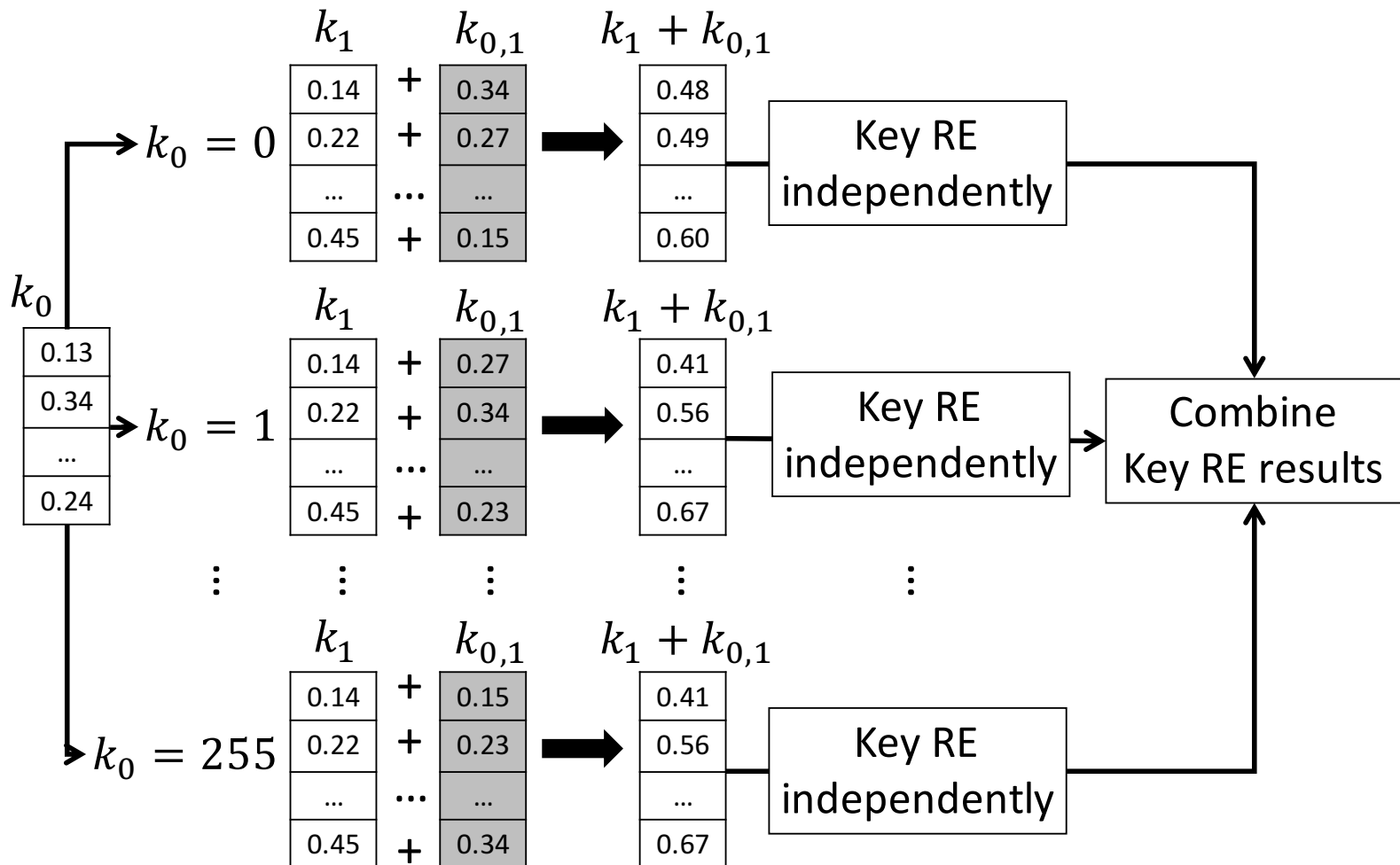
# Our Solution: divide and conquer

- Take  $k_0, k_1$  and  $k_{0,1}$  as an example, there is an XOR relationship between them as  $k_0 \oplus k_1 = k_{0,1}$
- When a subkey i.e.  $k_0$  is fixed, there is a one-one correspondence between  $k_1$  and  $k_{0,1}$
- We combine key lists of  $k_1$  and  $k_{0,1}$  to obtain a new list as  $k_1 + k_{0,1}$ , which is independent from  $k_2, k_3, \dots, k_{15}$



# Our Solution: divide and conquer

- Key RE for dependent lists is reduced to  $2^8=256$  key RE problems for independent key lists





# Our General Solution

- Fix 1 subkey  $k_i$ , there is correspondence between  $k_j$  and  $k_{i,j}$ ,  $j \in \{0,1, \dots, 15\}$ .
- Fix 1 key byte allows to add up to 15 subkey-difference score lists
- When  $k_i = m$ ,  $m \in \{0,1, \dots, 255\}$ , we combine  $(k_j, k_{i,j}) = s(k_j) + s(k_{i,j}=k_j \oplus m)$ ,  $s(k_j)$  represents the score of  $k_j$
- $k_p + k_{i,p}$  and  $k_q + k_{i,q}$  are independent from each other as well

$k_0 = 0$	$k_1 + k_{0,1}$	$k_2$	.....	$k_{15}$
0.13	0.48	0.14		0.29
	0.49	0.22		0.31
	...	...		...
	0.60	0.45		0.68

Adding 1 subkey-difference score list

$k_0 = 0$	$k_1 + k_{0,1}$	$k_2 + k_{0,2}$	.....	$k_{15} + k_{0,15}$
0.13	0.48	0.71		0.50
	0.49	0.85		0.66
	...	...		...
	0.60	0.83		1.45

Adding 15 subkey-difference score lists<sub>17</sub>

# Our General Solution

- If we want to add more than 15 dependent score lists, fix 1 subkey is not enough.
- We can fix 2 subkeys, such as  $k_0$ ,  $k_1$ , and add up to 29 lists
- $15+14=29$  lists
- There is a correspondence between  $k_0, k_i$  and  $k_{0,i}$  and  $k_{1,i}$ .

$k_0 = 0$	$k_1 = 0$	$k_2 + k_{0,2} + k_{1,2}$	$k_{15} + k_{0,15}$
0.13	0.14	1.03	0.83
		1.62	0.81
		...	...
		0.98	1.81

Adding 16 subkey-difference score lists

$k_0 = 0$	$k_1 = 0$	$k_2 + k_{0,2} + k_{1,2}$	$k_{15} + k_{0,15} + k_{1,15}$
0.13	0.14	1.03	0.83
		1.62	0.81
		...	...
		0.98	1.81

Adding 29 subkey-difference score lists

# Limitations of DK-REA

- Computational overhead
  - Adding  $N$  subkey difference lists
  - If  $1 < N < 16$ , computational complexity is increased by a factor of  $2^8$
  - If  $15 < N < 30$ , computational complexity is increased by a factor of  $2^{16}$
- Added dependent score lists are fixed
  - Dependent score lists that can be added cannot be freely chosen
  - Depends on the selected fixed subkey
  - e.g., when we select  $k_0$ , 15 dependent score lists that can be added are  $k_{0,1}, k_{0,2}, k_{0,3}, \dots, k_{0,15}$ .

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# Experimental Setup: simulated leakage

- Target: AES-128
  - 16 S-boxes in first AES round
  - Serial implementation
  - Leakage model: Hamming weight of the S-box output with noise
- We simulated two attacks
  - Correlation Power Analysis → key recovery
  - Correlation Enhanced Collision Attack → key difference recovery
- Two types of noises in the leakage measurement

$$LeakageModel(\alpha) = HW(\alpha) + \mathcal{N}(0, 4^2)$$

$$MesauredLeakage(\alpha) = LeakageModel(\alpha) + \mathcal{N}(0, 4^2)$$

# Experimental Setup: REA with independent score lists

- DK-REA uses a REA for independent score lists. All previous REA algorithms can be used here.
- In this paper, we use the key REA based on histograms for its simplicity and efficiency.

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**Algorithm 2** Rank estimation algorithm based histograms [9]

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**Input:** The score lists and the histograms  $H_i$

**Output:** An approximation of the rank of the correct key.

1: Initialization:  $H_{curr} = H_1$ ;

2: Histograms convolution:

3: **for**  $i = 2 : N_p$  **do**

4:      $H_{curr} = conv(H_{curr}, H_i)$ ;

5: **end for**

6: Rank estimation:

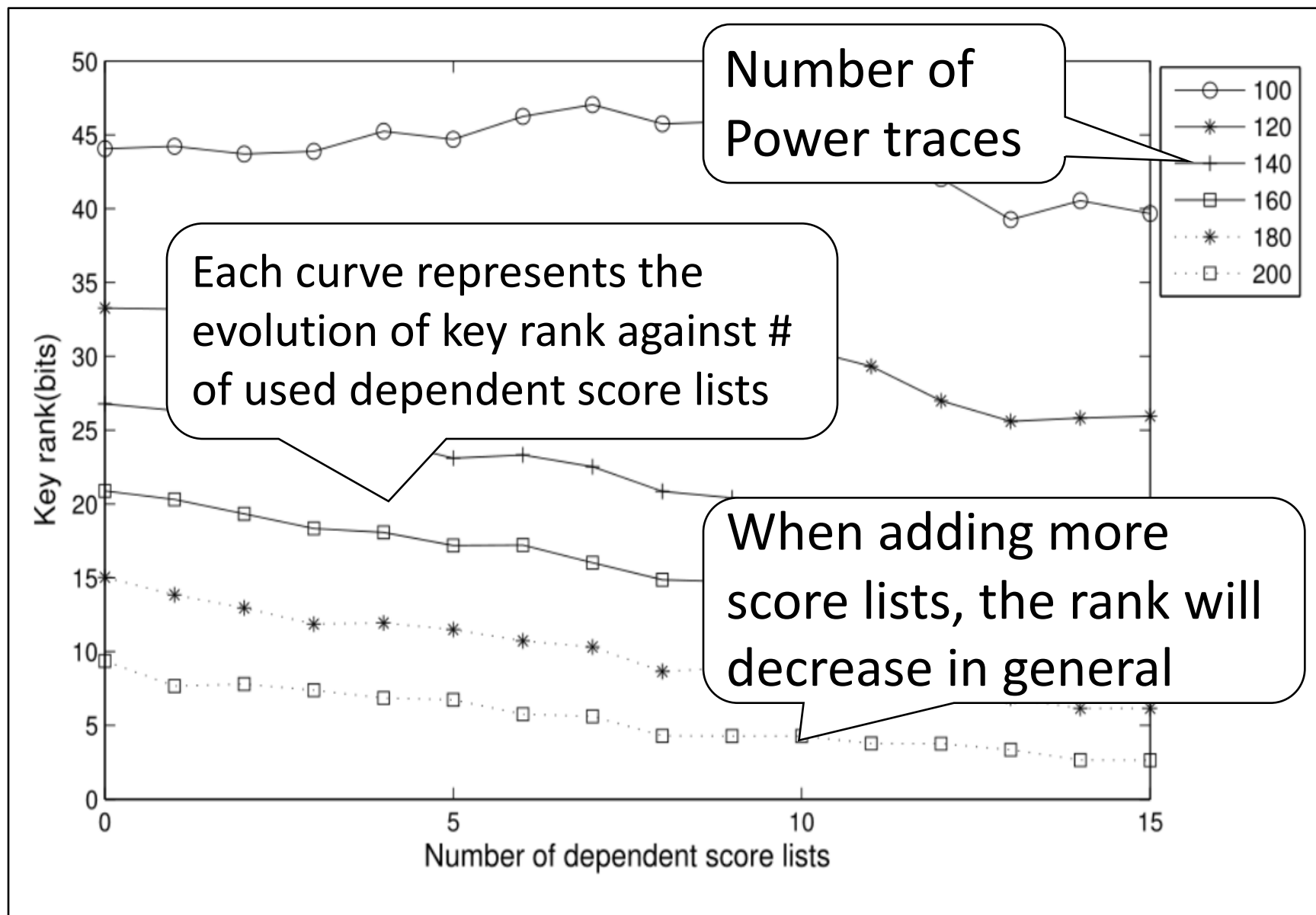
7:  $Estimated\_rank \approx \sum_{i=bins(k)}^{N_p \cdot N_{bin} - (N_p - 1)} H_{curr}(i)$

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# Experimental Setup: repetitions

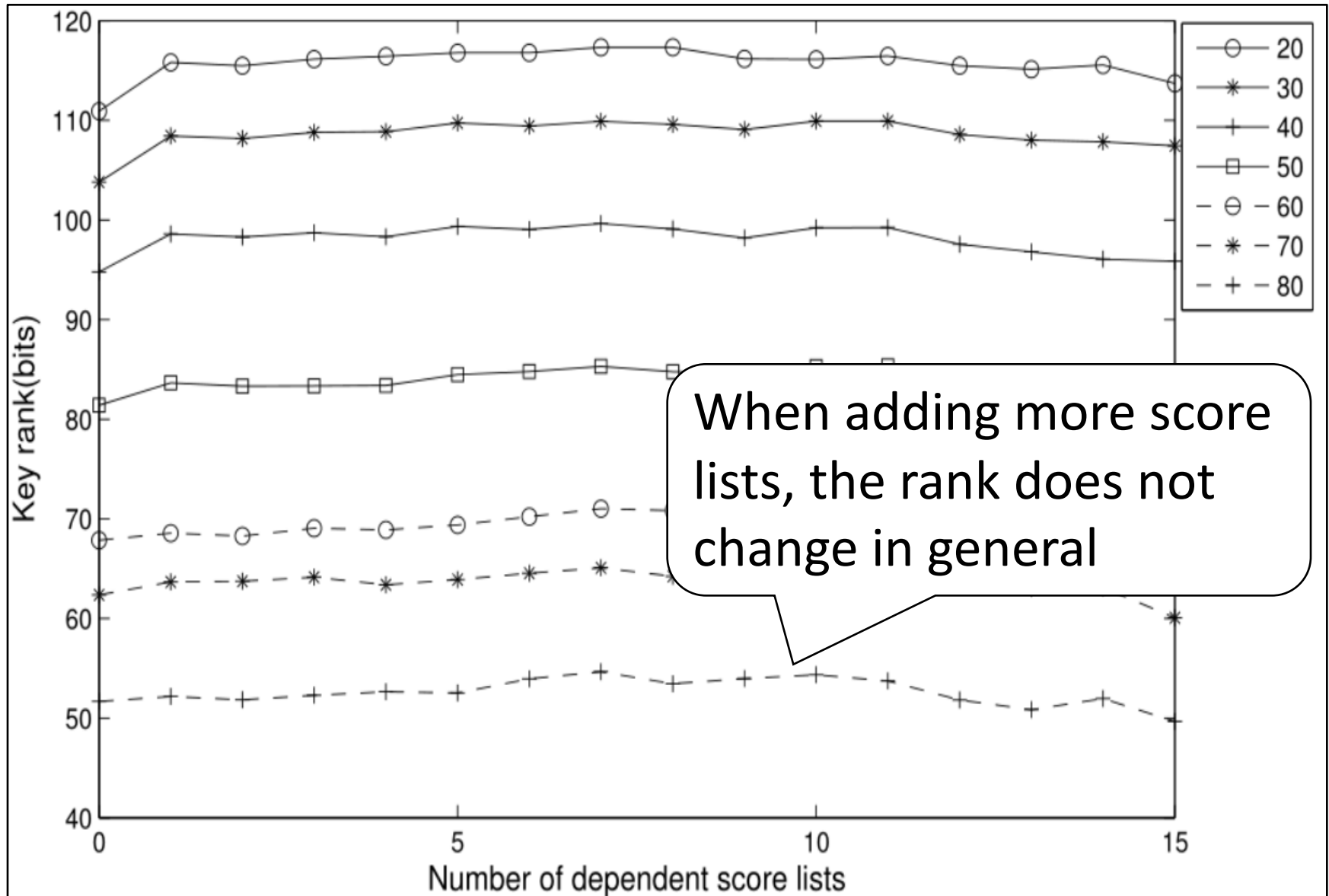
- We performed
- 100 times experiments for adding 0 to 15 subkey-difference score lists
- 10 times experiments for adding 0 to 29 subkey-difference score lists.
- Taking into account the constraints of time and space, we do not consider the key rank estimation using more than 29 subkey-difference score lists

# DK-REA up to 15 lists (100+ traces)

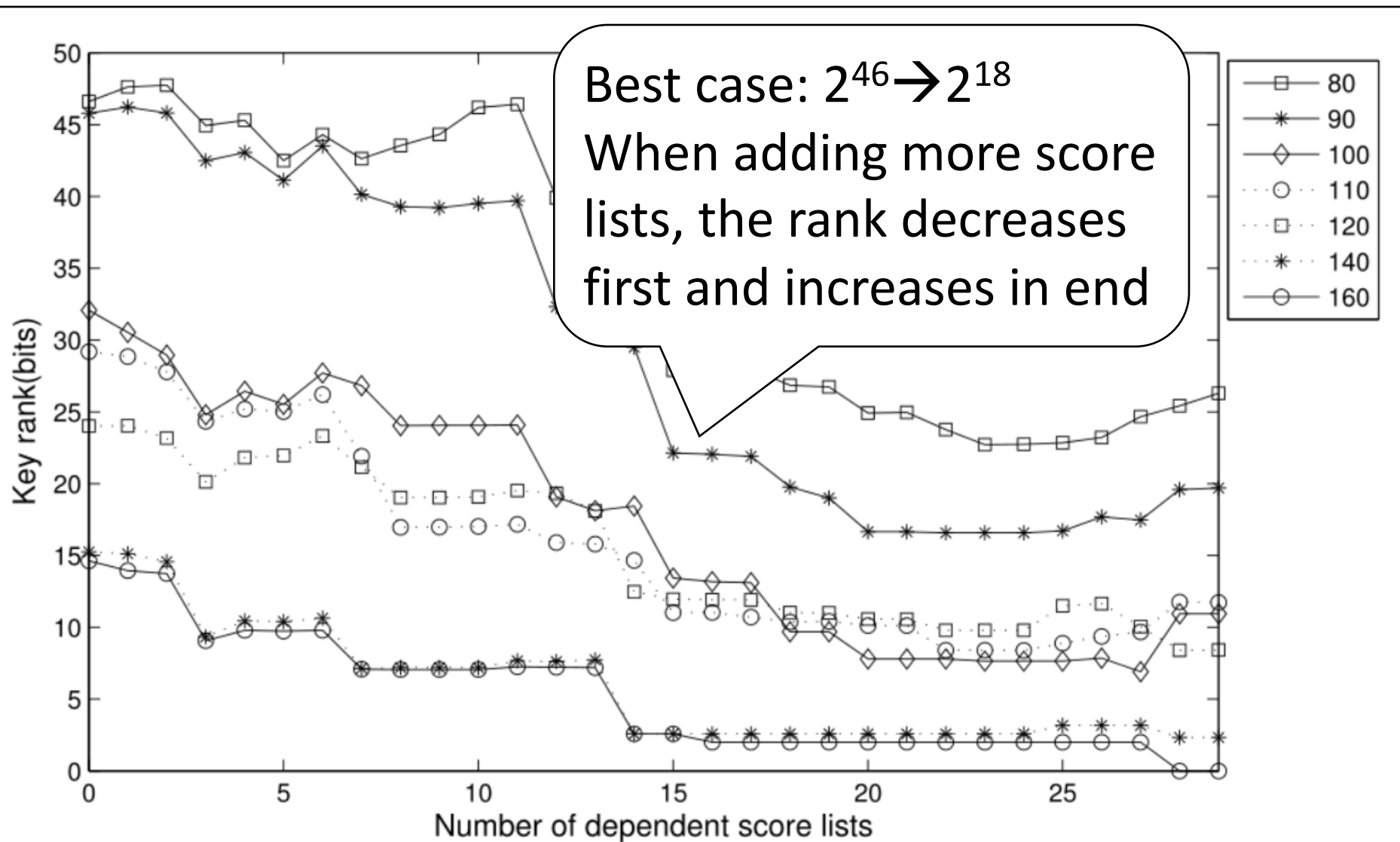




# DK-REA up to 15 lists (<100 Traces)



# DK-REA up to 29 lists



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# Conclusions

- In this paper, we proposed DK-REA
  - Able to estimate key rank for dependent score lists
  - Generally the attack result is improved with more lists
    - Key rank reduced from  $2^{46}$  to  $2^{18}$  in the best case
  - Improvement also has limitations
    - When data is not enough
    - When too many dependent lists are added
- DK-REA is
  - A new tool in SCA to explore the possibility of merging various attack results for a more accurate security evaluation result

# Future work

- More experiments to verify the effects of DK-KEA
  - With practical power traces
  - With different structures
    - e.g. serial and parallel implementations
  - With different leakage models
    - e.g. two attacks with large difference in key recovery efficiency
- DK-KEA for more complex dependency relations such as algebraic side-channel attack
- Reduce overhead!
- Rank Estimation Alg. → Key Enumeration Alg.
  - Our work to be presented at CARDIS 2017 in this Nov.

Thanks very much for  
your attentions!

Questions?

[li.yang@nuaa.edu.cn](mailto:li.yang@nuaa.edu.cn)

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