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A Practical Cold Boot Attack on RSA Private Keys

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AsianHOST 2017

Outline

- Cold boot attacks on RSA encryption schemes

-Cold boot attack and memory decay

-Existing algorithms to recover RSA private keys

- Practical cold boot attacks on RSA private keys

-Limitations of the existing algorithms

-Our improvements

- Evaluation results and discussion
- Conclusion

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Cold boot attack

- J. Halderman et al, USENIX Security Symposium, August 2008
- Memory remanence effect -Data persists after power-off.
- Memory blocks are transferred to other machines to extract the encryption keys .

Fig.1 A typical cold boot attack[1]

[1] J.A. Halderman et al, “Lest we remember: cold boot attacks on encryption keys,” in *USENIX Security Symposium*, 2008, pp. 45–60

Memory decay

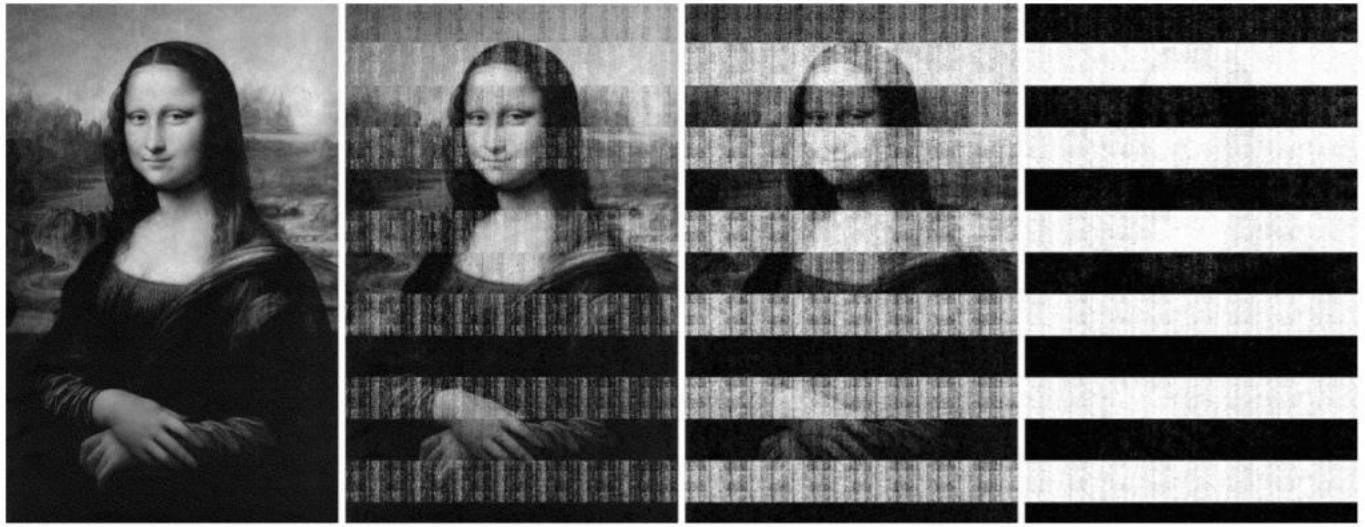

Fig.2 Visualizing memory decay[1]

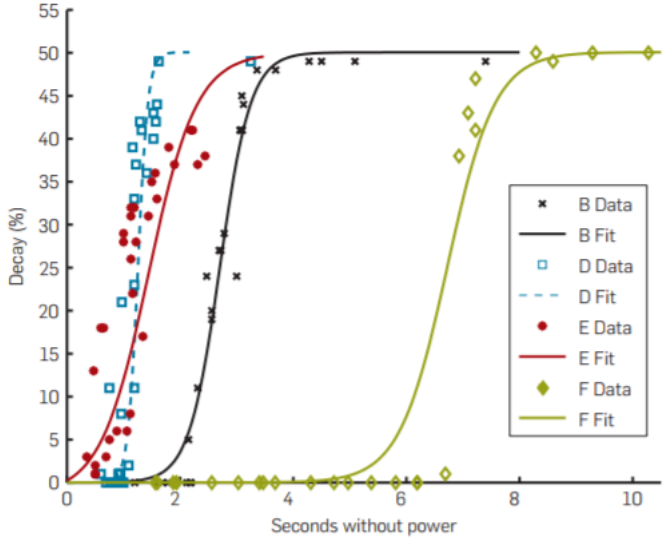

Fig.3 Measuring decay for different test systems[1]

Bit flip!

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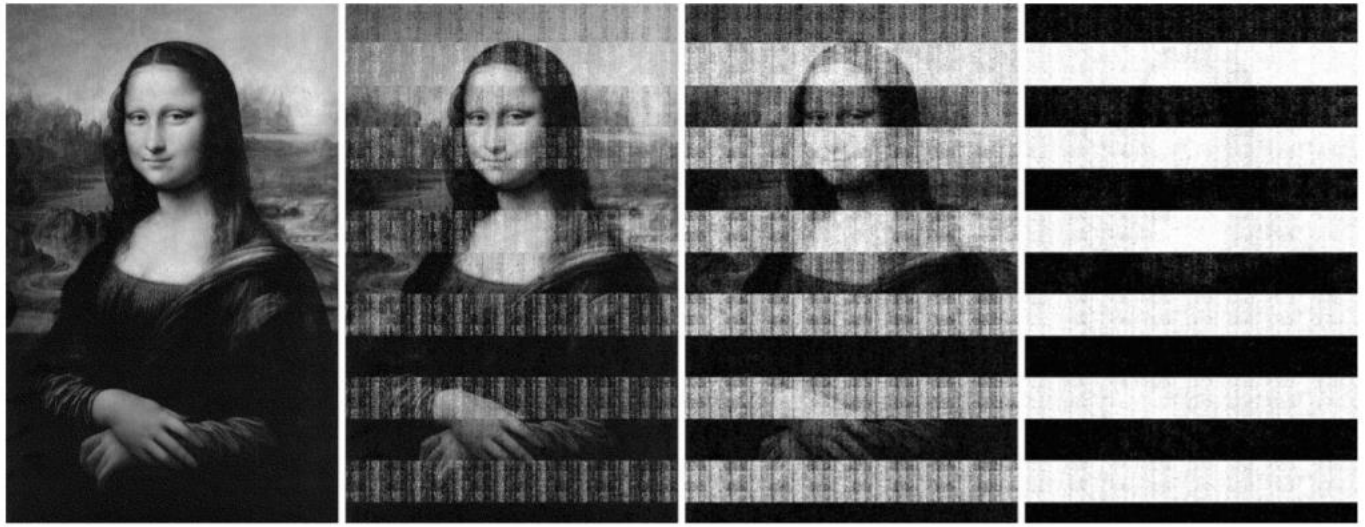

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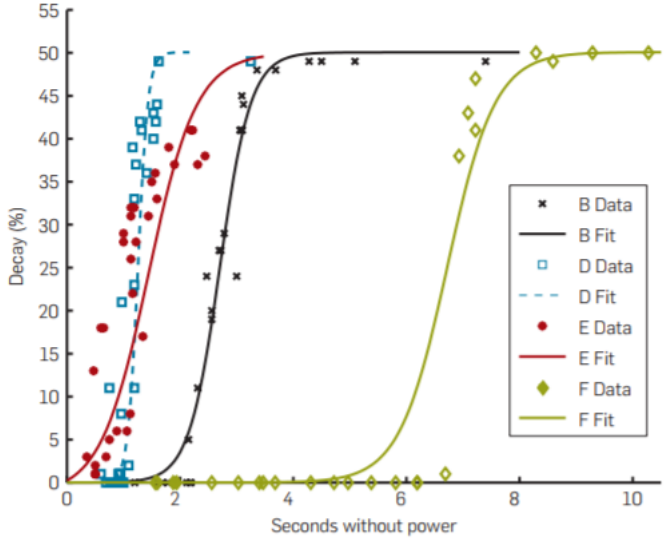
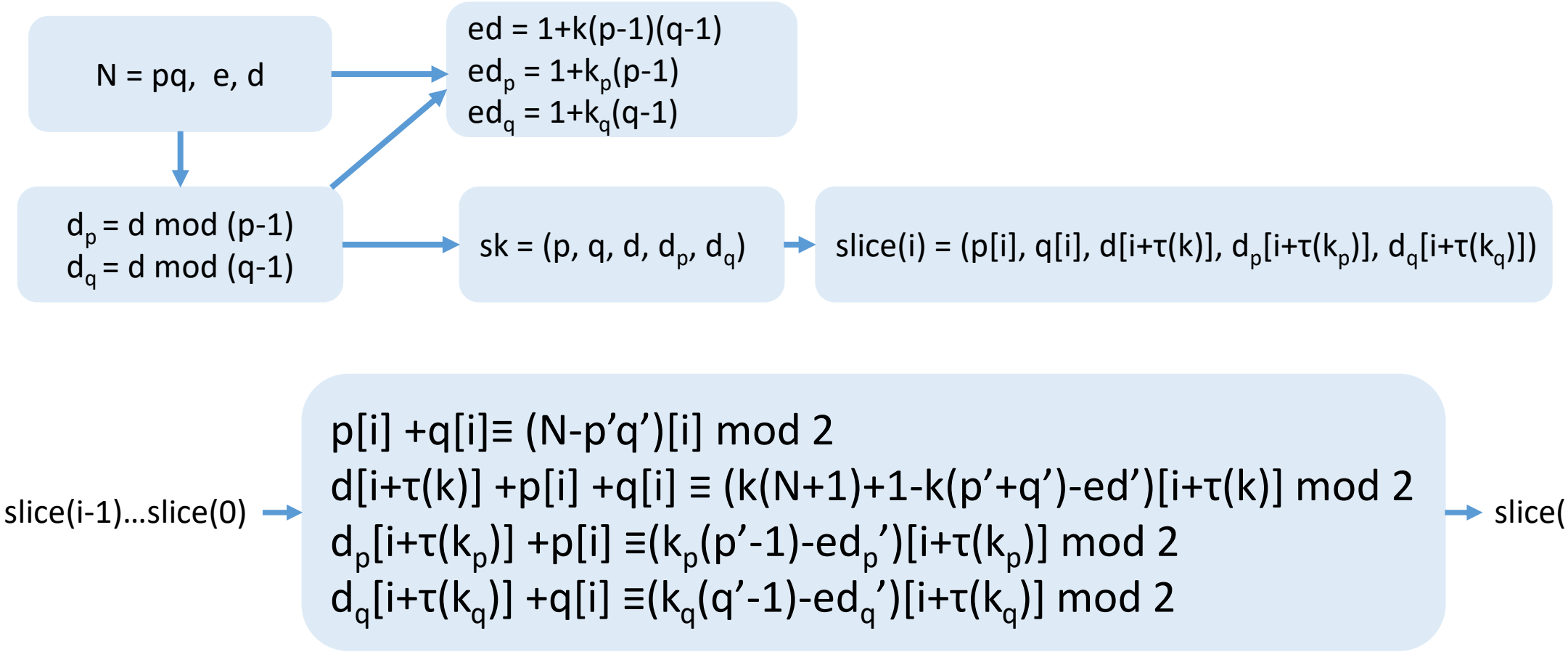


Fig.3 Measuring decay for different test systems[1]

- Cooling
- Error correction method

[1] J.A. Halderman et al, “Lest we remember: cold boot attacks on encryption keys,” in *USENIX Security Symposium*, 2008, pp. 45–60

Existing algorithms to recover RSA private keys(1/4)



Existing algorithms to recover RSA private keys(2/4)

- Hensel lifting

$p[i] + q[i] \equiv c_1$

$d[i+\tau(k)] + p[i] + q[i] \equiv c_2$

$d_p[i+\tau(k_p)] + p[i] \equiv c_3$

$d_q[i+\tau(k_q)] + q[i] \equiv c_4$

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Slice[0]

$p[0] = 1$
$q[0] = 1$
$d[\tau(k)]\ldots d[0]$
$d_p[\tau(k_p)]\ldots d_p[0]$
$d_q[\tau(k_q)]\ldots d_q[0]$

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$d_q[\tau(k_q)]\ldots d_q[0]$

Slice[1]

$p[1] = 1$
$q[1] = 1-c_1$
$d[\tau(k)+1]=c_2-c_1$
$d_p[\tau(k_p)+1]=c_3-1$
$d_q[\tau(k_q)+1]=c_4-q[1]$

Existing algorithms to recover RSA private keys(2/4)

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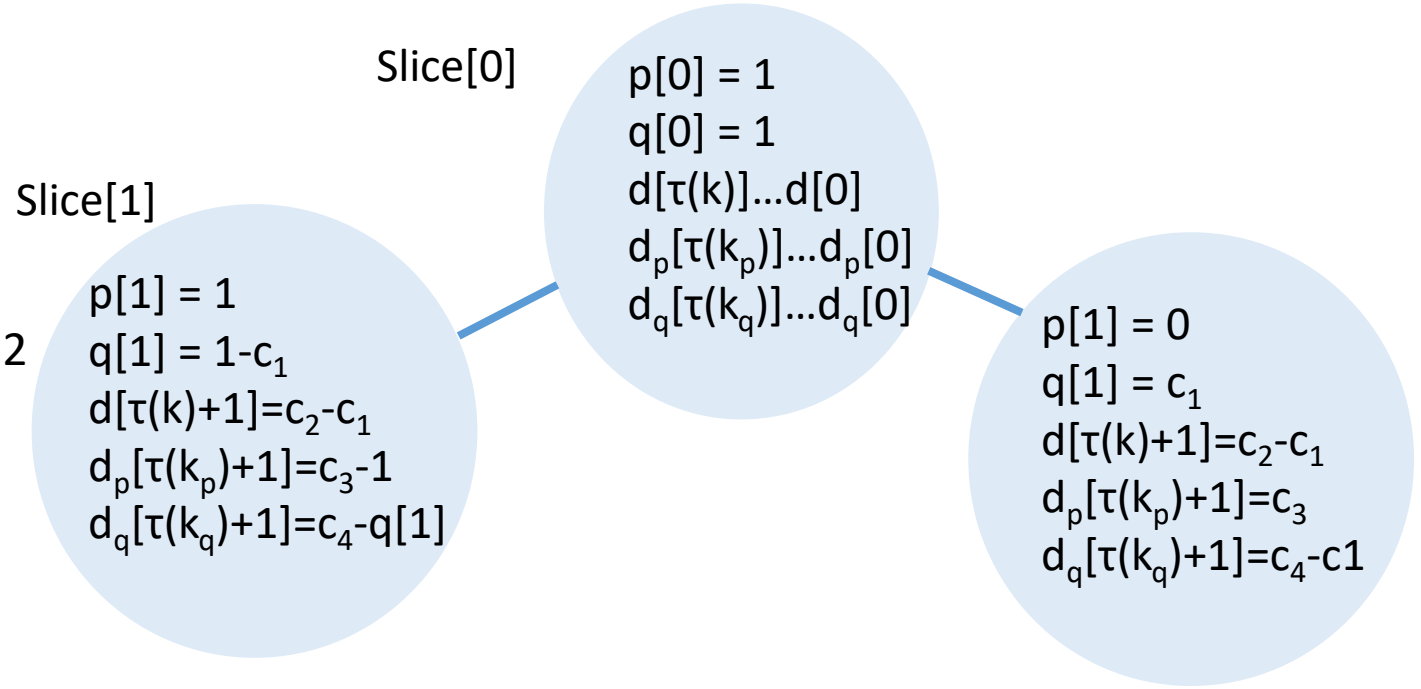

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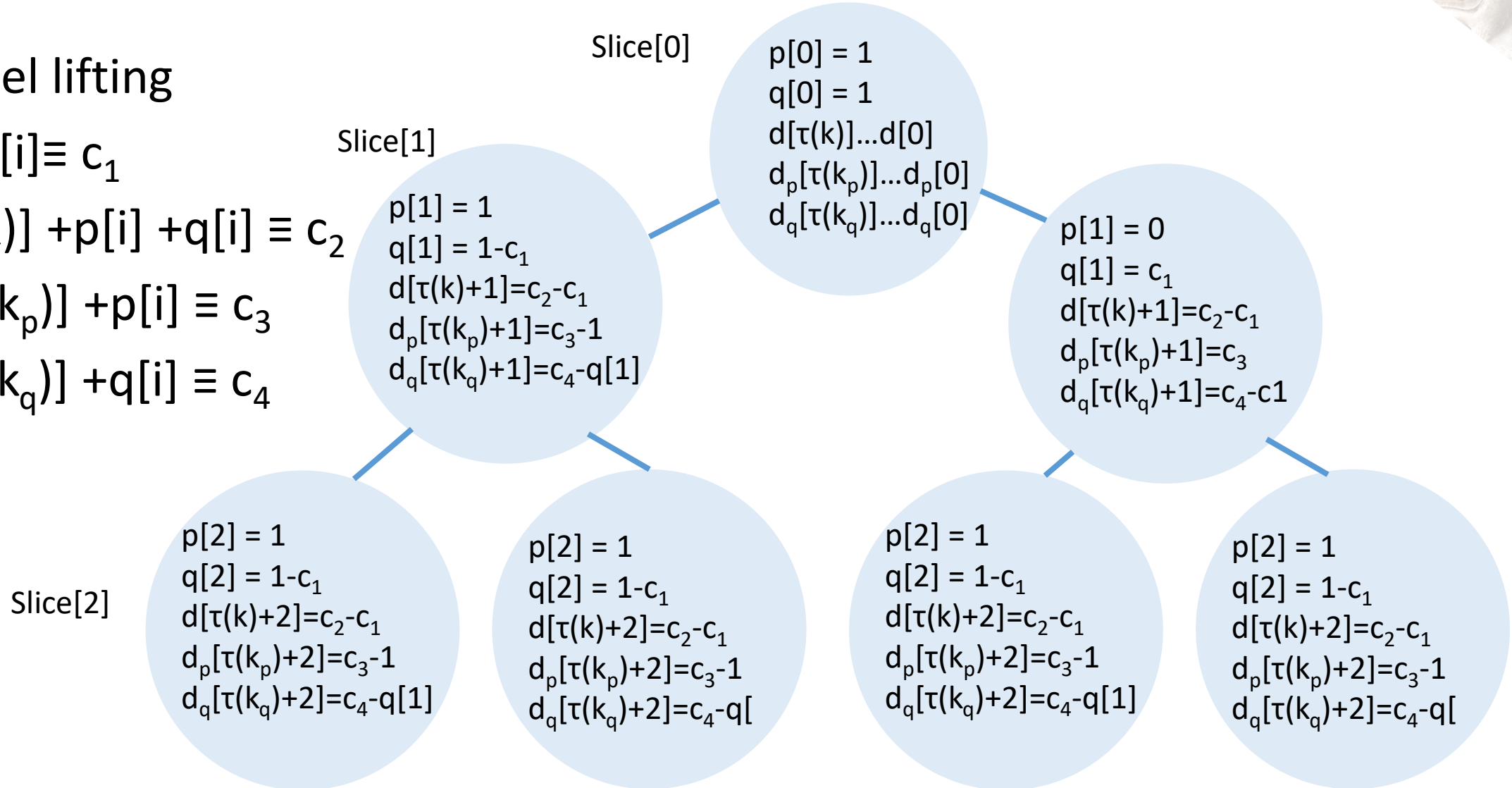
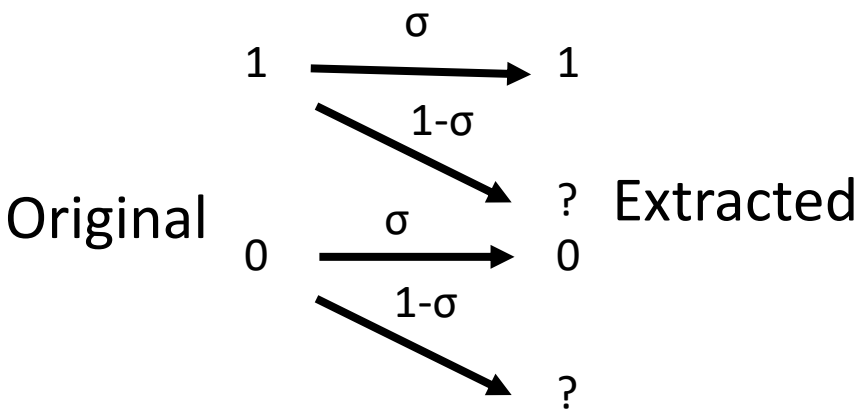
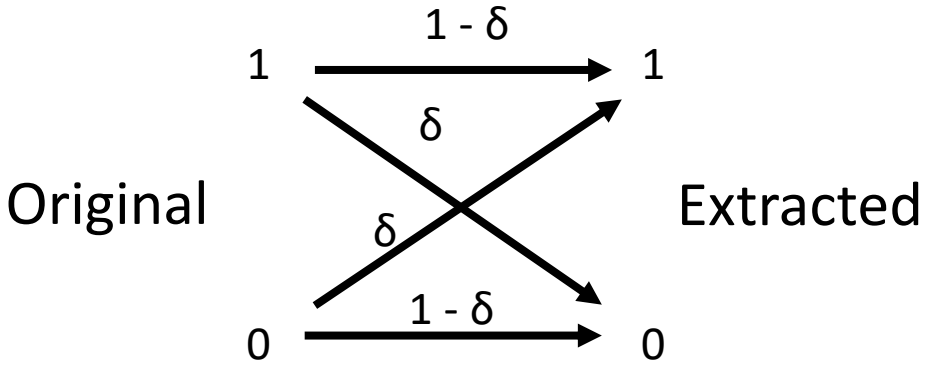


Existing algorithms to recover RSA private keys(3/4)

HS algorithm[2]	HMM algorithm[3]
A fraction σ(σ>0.27) of the extracted bits are known.	There exist a flipping probability δ(δ<0.237) for each bit.





[2] N. Heninger and H. Shacham, "Reconstructing RSA private keys from random key bits," *Advances in Cryptology-CRYPTO 2009.* Springer Berlin Heidelberg, 2009, pp.1-17
[3] W. Henecka, A. May and A. Meurer, "Correcting errors in RSA private keys," *Advances in Cryptology–CRYPTO 2010.* Springer Berlin Heidelberg, 2010, pp.351-369.

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HS algorithm

HMM algorithm

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Expansion

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Pruning		

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Existing algorithms to recover RSA private keys(4/4)

HS algorithm

Expansion: Build candidates with Hensel lifting **1** bit a time

Pruning: Remove the candidates that don't match with known key bits

HMM algorithm

Build candidates with Hensel lifting **t** bits a time

Calculate the number of matching bits

Existing algorithms to recover RSA private keys(4/4)

HS algorithm

Expansion

Build candidates with Hensel lifting **1** bit a time

Pruning

Remove the candidates that don't match with known key bits

HMM algorithm

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Calculate the number of matching bits

Calculate threshold **C**

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Remove the candidates that don't match with known key bits

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Build candidates with Hensel lifting **t** bits a time

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Calculate threshold **C**

Remove the candidates whose number of matching bits is less than the threshold **C**

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HMM algorithm

If $\delta \le 0.5 - \gamma - \varepsilon$
$t = [\ln 2n / 10\varepsilon^2]$
$\gamma = (\ln 2 (t+1)/10t)^{0.5}$
$C = 5t(0.5+\gamma)$

Build candidates with Hensel lifting **t** bits a time

Calculate the number of matching bits

Calculate threshold **C**

Remove the candidates whose number of matching bits is less than the threshold **C**

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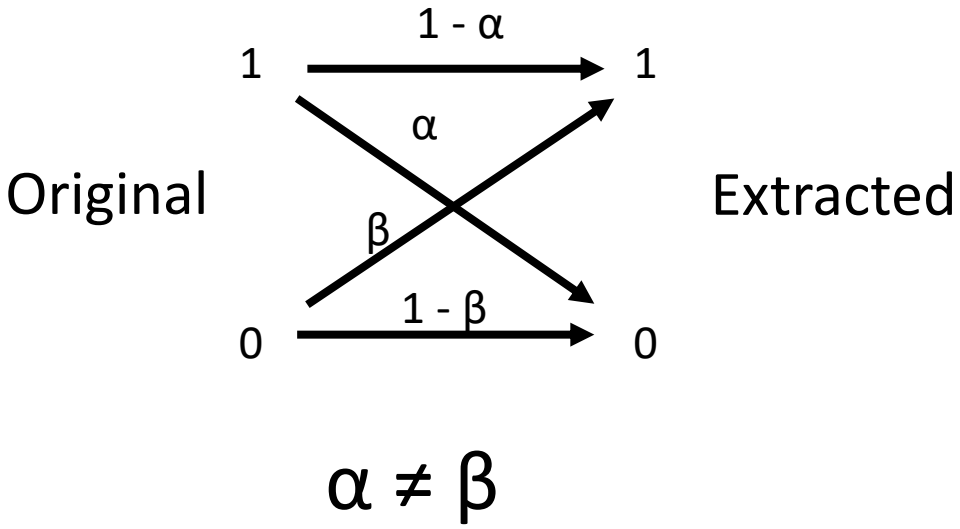
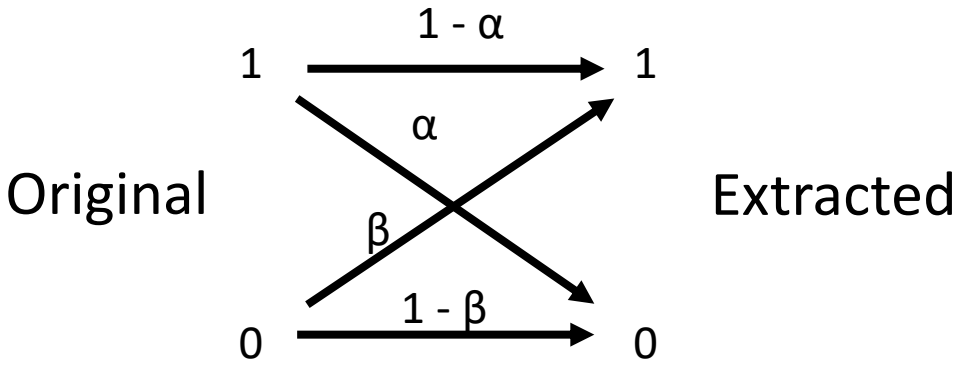


$$\alpha \neq \beta$$

Our improvements based on HMM algorithm(1/4)

- Real-life settings.
- Asymmetric pruning strategy.



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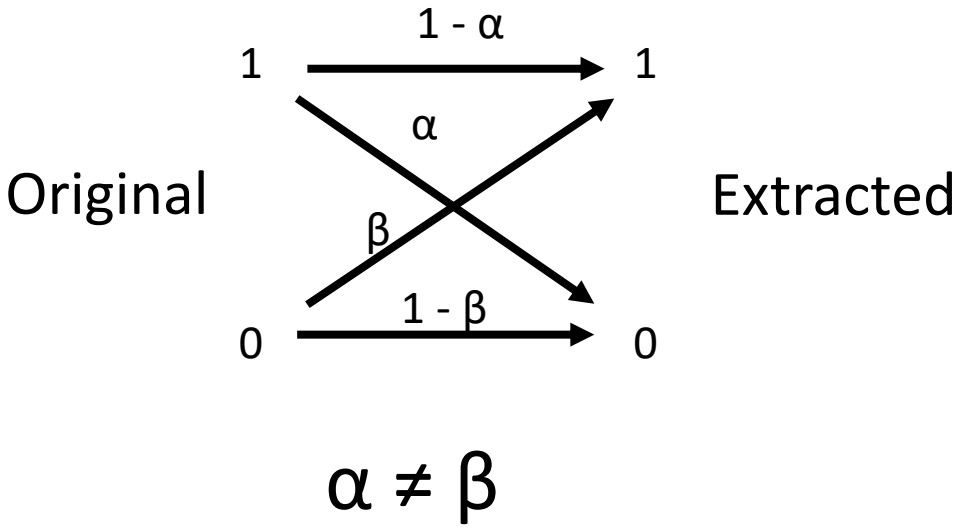

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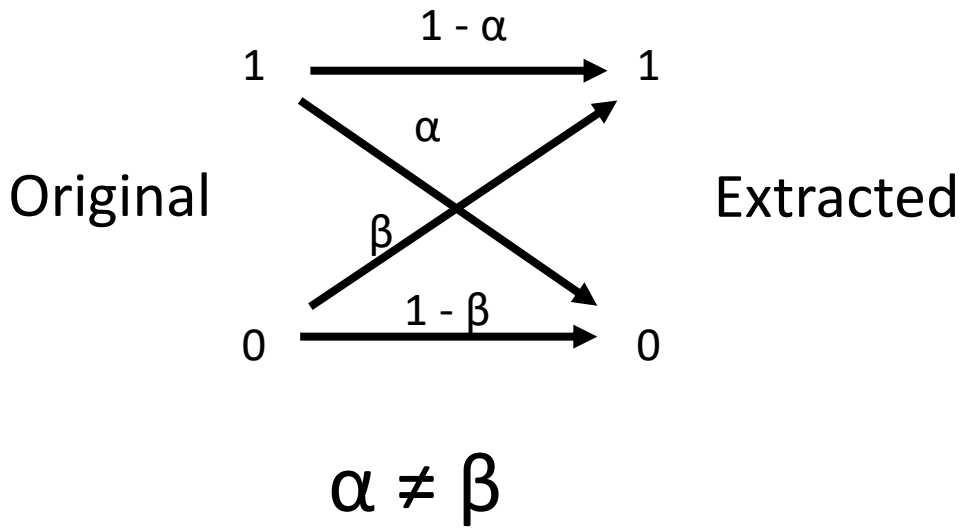

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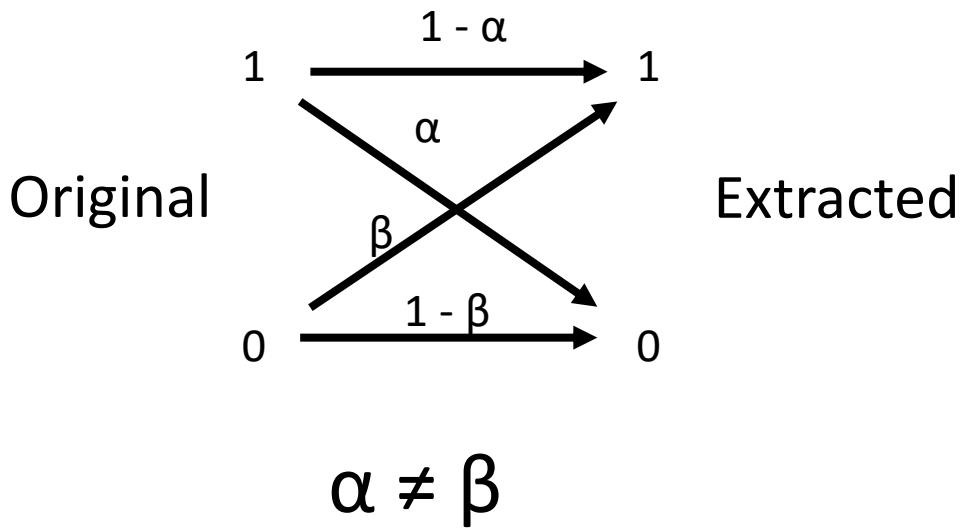

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Build candidates with Hensel lifting **t** bits a time

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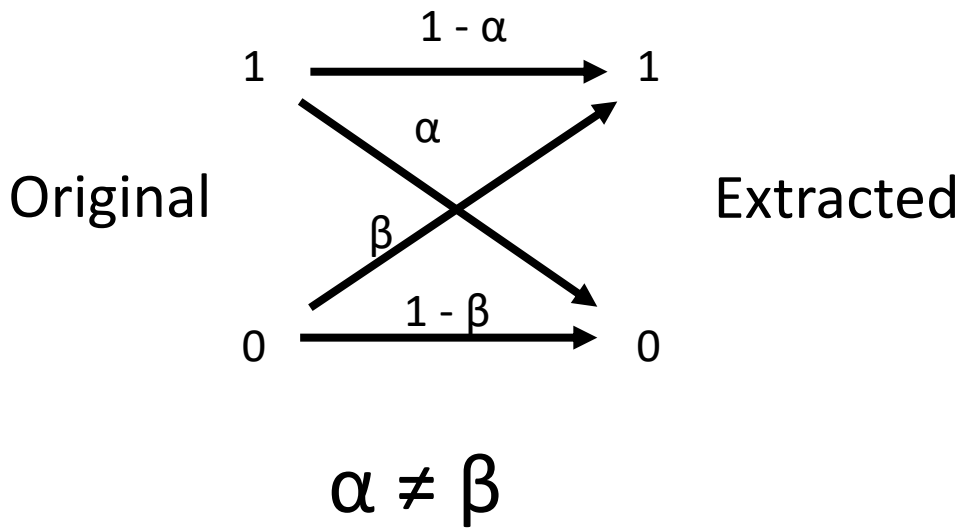

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Build candidates with Hensel lifting **t** bits a time — Expansion

Pruning

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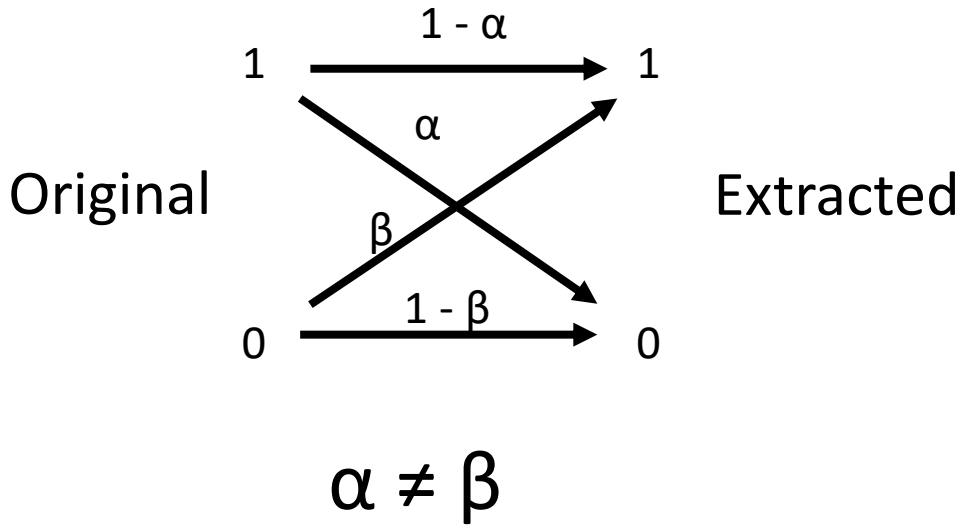

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Build candidates with Hensel lifting **t** bits a time — Expansion

Calculate the number of matching bit 0 and matching bit 1 respectively — Pruning

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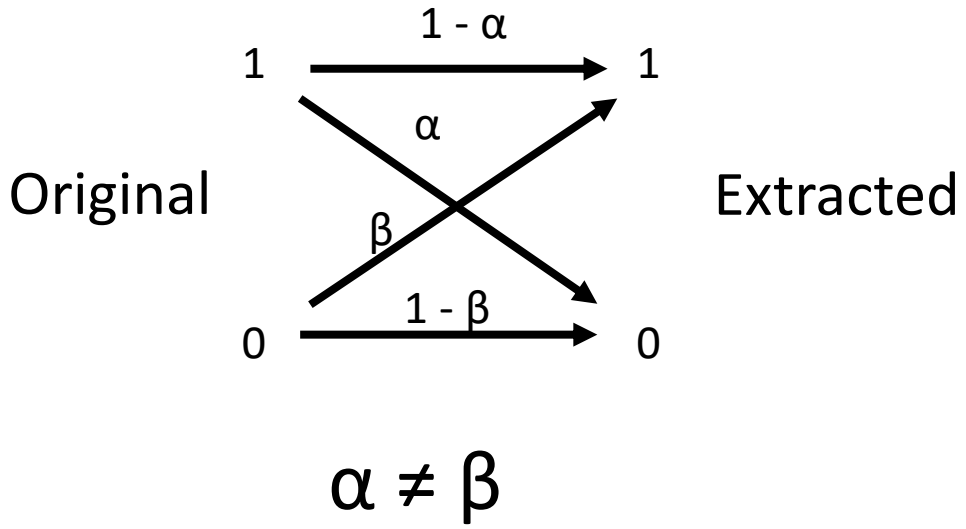

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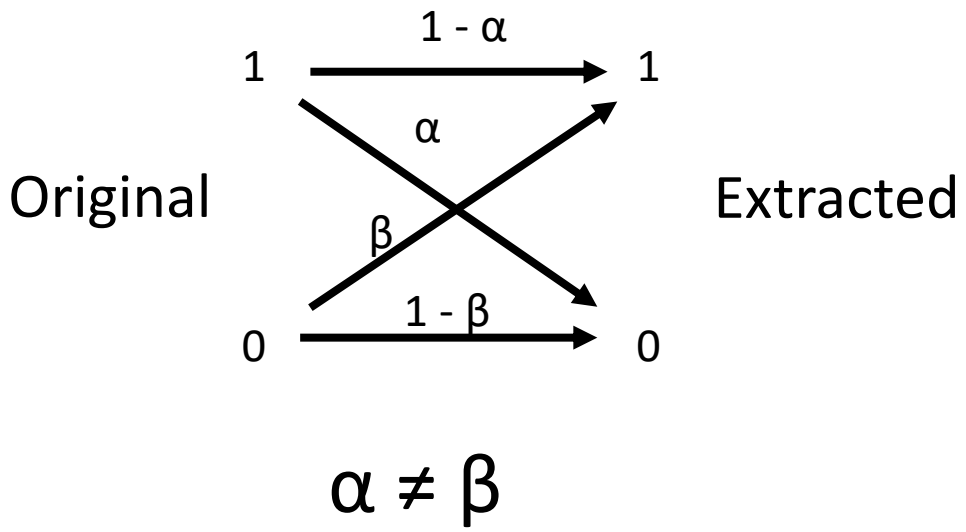

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Build candidates with Hensel lifting **t** bits a time — Expansion

Calculate the number of matching bit 0 and matching bit 1 respectively — Pruning

Calculate two thresholds for bit 0 and bit 1 $\mathbf{C_0}$ and $\mathbf{C_1}$

Remove the candidates whose number of matching bit 0 (or 1) is less than the corresponding threshold $\mathbf{C_0(C_1)}$

Our improvements based on HMM algorithm(1/4)

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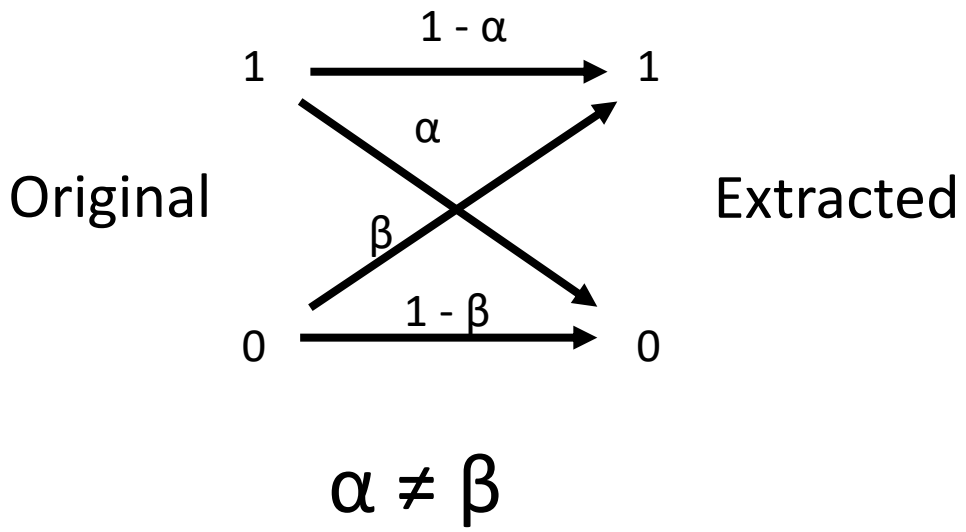

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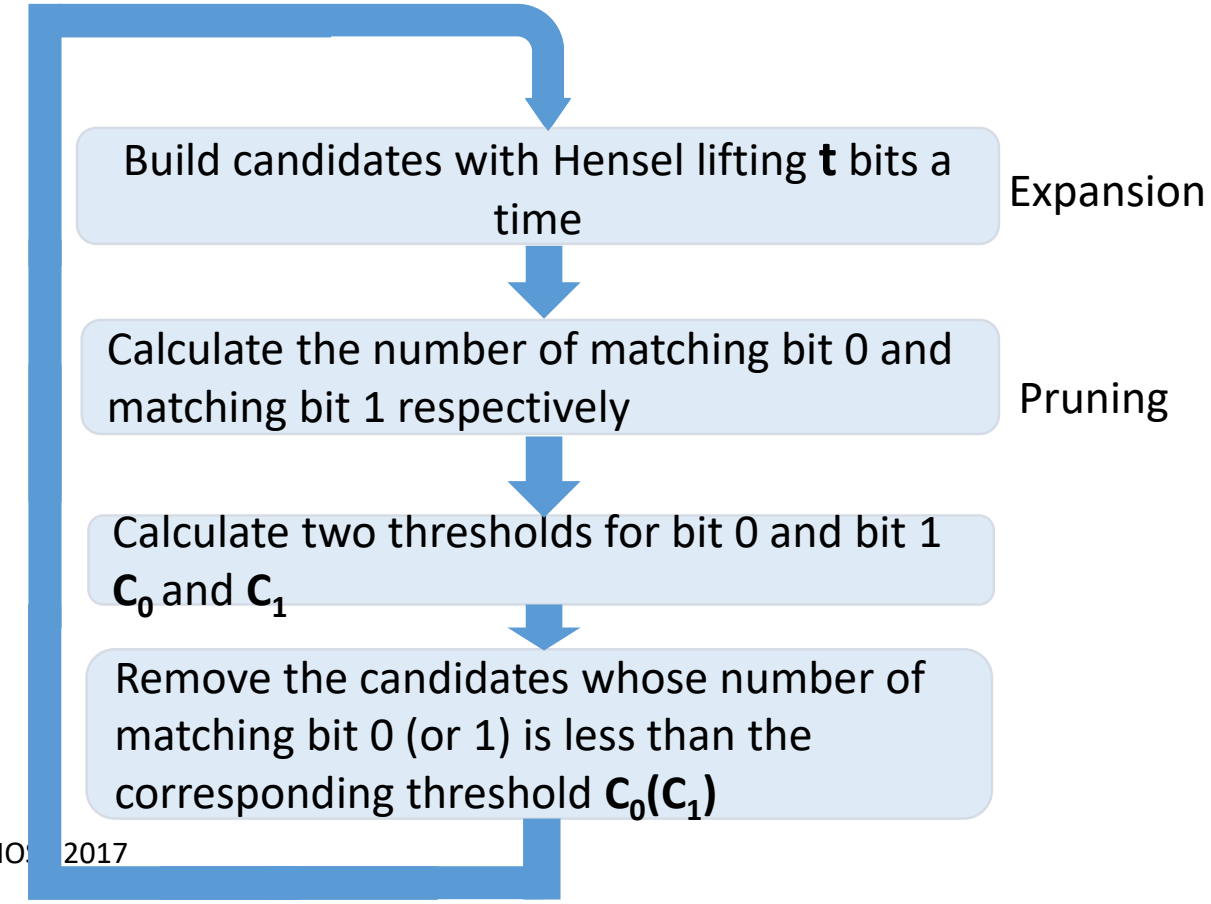

Our improvements based on HMM algorithm(2/4)

The Improved HMM Algorithm
Input: (N,e), erroneous sk, error probabilities α and β
Output: the correct sk.
Initialization: Calculate (k, k_p, k_q) and slice(0)
For round = 1 to n/(2t)+1 Expansion: Expand t times for every candidate with slice(0)…slice(round-1)t using the Hensel lifting, which generates 2^t different partial candidates gc_1, $gc_{2\ldots\ldots}$ gc_2t in slice((round-1)t+1)… slice(round t).

Pruning:

For i = 1 to 2^t
--Count the number of matching bit 0(1) $X_{i0(1)}$ of gc_i in slice((round-1)t+1)… slice(round t) with the given erroneous sk.
--Compute the threshold $C_{i0(1)}$ in the ith round.
If (X_{i0} <= C_{i0} or X_{i1} <= C_{i1}) Discard the candidate.
Else Keep the candidate
end for
end for

Finalization: Run trial decryption to determine the correct private key.

Our improvements based on HMM algorithm(3/4)

- Notation

For any ε_0 ε_1>0, (N,e) as the n-bit modulus and public key.

Given flipping probability α, β, η =min((1- α) ε_1^2, ε_0^2)

$p_{1(0)}$: the error probability of bit 1(0) in the extracted key

$N_{0(1)}$: the number of bit 0 (1) in each *t* slices of the extracted key

$N_{0(1)}$': the number of bit 0 (1) in each *t* slices of the candidate key

- Choice of parameters

We choose t, t_0, t_1 so that t = [ln 2n /5η] $t = 1+t_0+t_1$

Our improvements based on HMM algorithm(4/4)

- Performance analysis

$\gamma_1 = (\ln 2\ (t_1+1)/2N_1)^{\wedge}0.5$ $\gamma_0 = (\ln 2\ (t_0+1)/2N_0')^{\wedge}0.5$

$C_1=N_1(0.5+\gamma_1)$ $C_0=N_0'(N_0/5t+\gamma_0)$

If $p_1 \leq 0.5-\gamma_1-\varepsilon_1$, $\beta \leq 1-N_0'-\gamma_0-\varepsilon_0$

The algorithm can output the correct sk in polynomial time **$O(n^{2+\ln 2/2.5\eta})$**

with success probability greater than $1-5\eta/2\ln(2n)-1/n$.

*success probability: The possibility that correct candidate isn't pruned.

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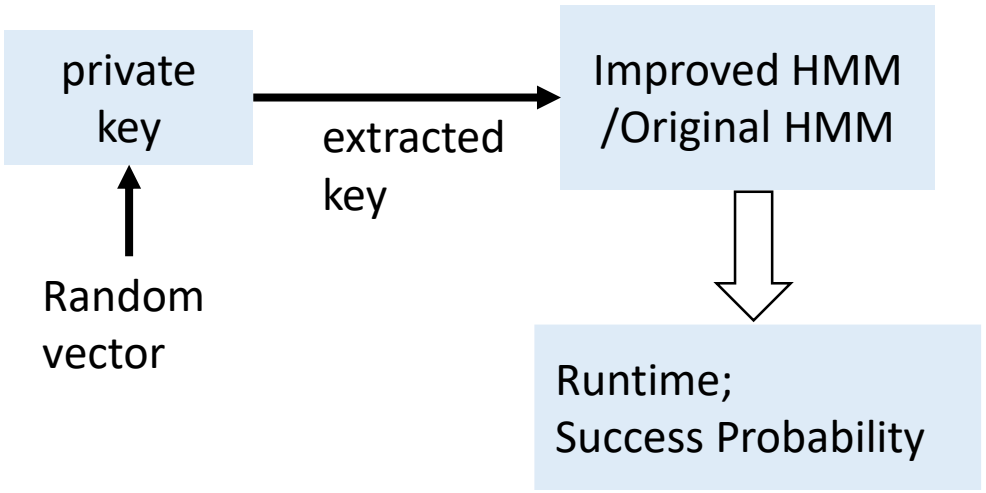
Evaluation results(1/3)

- $e = 2^{16}+1$
- $0.05 \le \alpha \le 0.4$ $\beta = 0.001, 0.01$
- 100 different 1024-bit private keys
- 100 repetitions for each private key



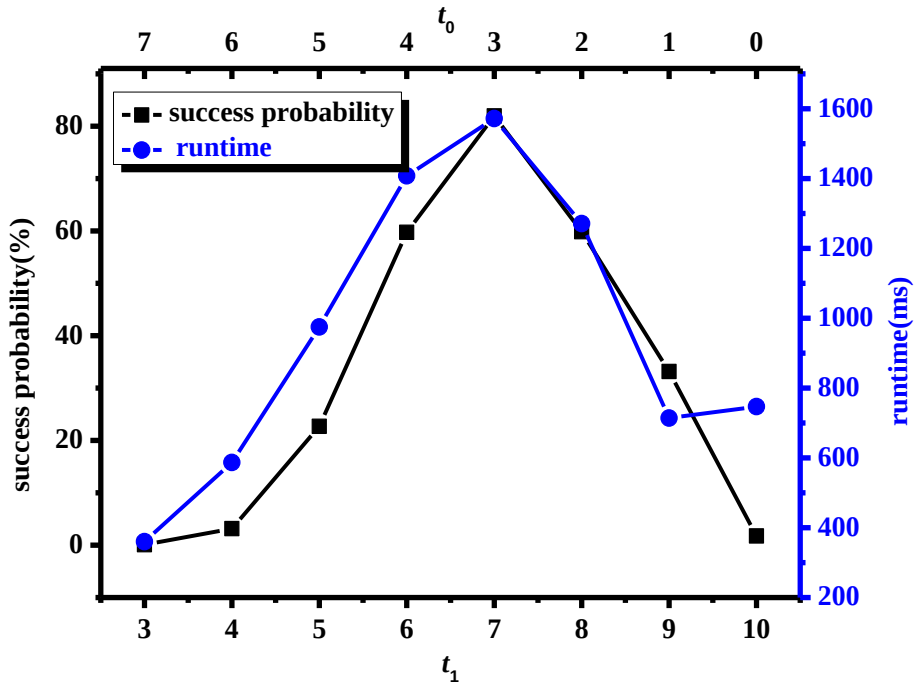


Fig. 4 The success probability and runtime for different combinations of (t_0,t_1) when $\alpha = 0.25$, $\beta = 0.001$ and $t = 11$.

Evaluation results(2/3)

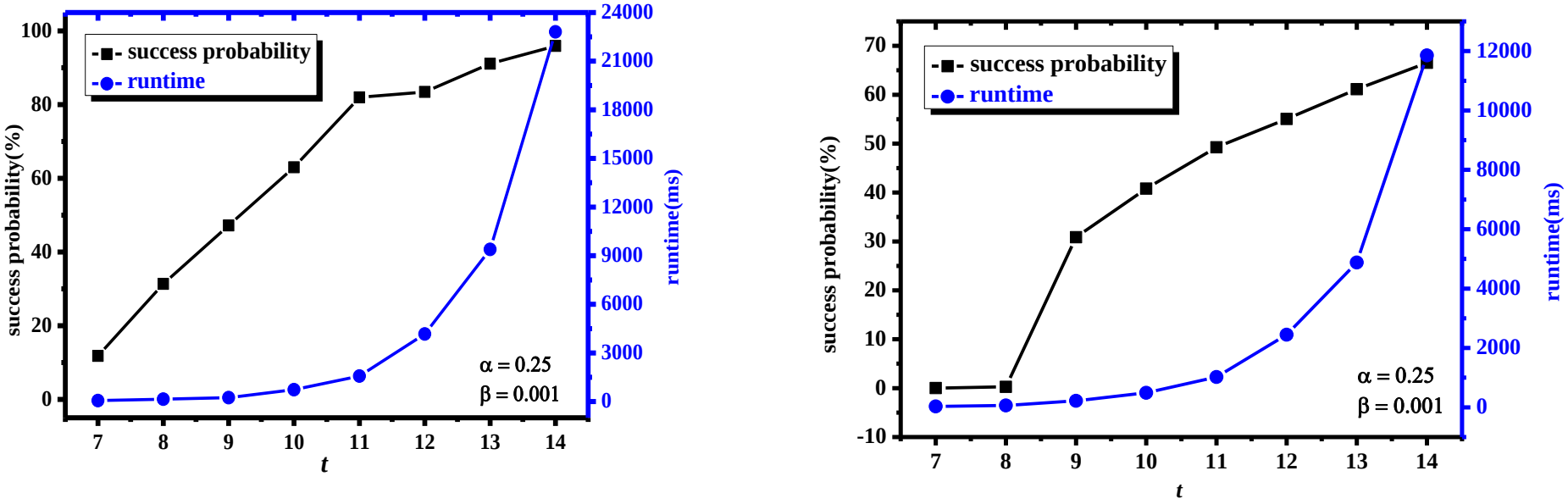


Fig. 5 Success probability and runtime of the improved HMM algorithm(a) and the original HMM algorithm(b)

Optimized t!

Evaluation results(3/3)

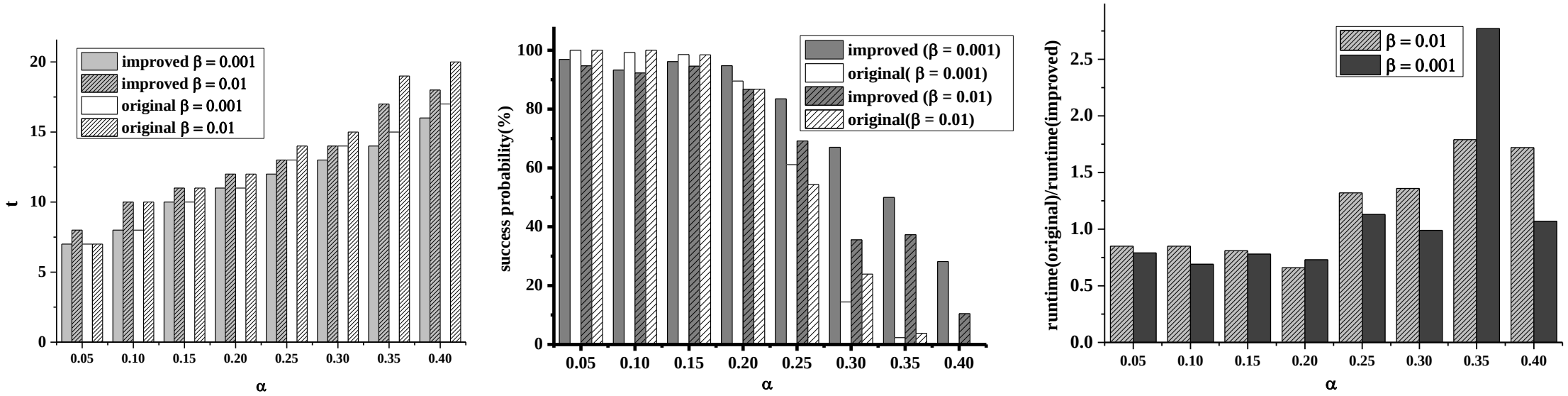


Fig. 5 Success probability and runtime of the improved HMM algorithm and the original HMM algorithm

- $\alpha\uparrow$ or $\beta\uparrow$ -> $t\uparrow$ -> success probability $\downarrow$
- $\alpha < 0.2$
- $\alpha > 0.2$

Discussion

- $C = C_1' + C_0' = N_0'(0.5+\gamma) + (5t - N_0')(0.5+\gamma)$
- $C_1 = N_1(0.5+\gamma_1)$ $\quad C_0 = N_0'(N_0/5t+\gamma_0)$
- $\alpha < 0.2 \quad C_1' \approx C_1 \quad C_0' < C_0$

$$P(X_{c0} + X_{c1} > C_0' + C_1') > P(X_{c0} + X_{c1} > C_0 + C_1)$$

$$> P(X_{c0} > C_0 \cap X_{c1} > C_1)$$

- $\alpha > 0.2 \quad C_1' > C_1$

$$P(X_{c0} + X_{c1} > C_0' + C_1') < P(X_{c0} > C_0 \cap X_{c1} > C_1)$$

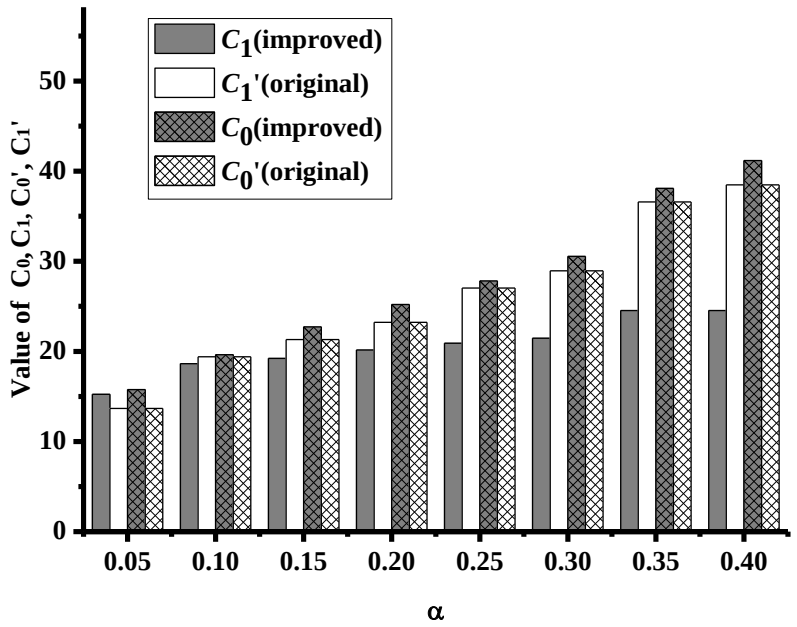


Fig. 6 Estimated threshold paramters of the improved and original HMM algorithm for different error probabilities α when β = 0.001

Outline

- Cold boot attacks on RSA encryption schemes

-Cold boot attack and memory decay

-Existing algorithms to recover RSA private keys

- Practical cold boot attacks on RSA private keys

-Limitations of the existing algorithms

-Our improvements

- Evaluation results and discussion
- *Conclusion*

Conclusion

- A practical improvement of the cold boot attack based on HMM algorithm

  -Real life settings

  -Asymmetric pruning strategy

- Performance analysis

Thank you!